SE(EXTC) Sem-3 C-Scheme Winter 2025 Date: 10/11/25

Time: 3 hour **Max. Marks: 80**

Note: 1) Question 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

3) Figures to the right indicate full marks.

Q1 Attempt All questions

A	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & -2 \end{bmatrix}$ then find the eigen values of A^2	5
B	Find Laplace transform of $f(t) = te^t \cos 2t$	5
C	Find the half range Sine Series for $f(x) = x$, where $x \in (-\pi, \pi)$	5
D	Determine the constant a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic.	5

Q2

A	Using Green's theorem in a plane to evaluate the line integral $\oint_C (xy + y^2)dx + x^2 dy$ Around the boundary of the region defined by $y=x^2$ and $y=x$	6
B	Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$	6
C	Show that the function $v = e^x siny$ satisfies Laplace's equation, also find analytic function.	8

Q3

A	Prove that $\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational.	6
B	Find the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$	6
C	Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$	8

Q4

A Using Stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ 6

Where $\bar{F} = x^2 i + xyj$ where C is the boundary of the rectangle x=0, y=0, x=a, y=b

B Evaluate $\int_0^\infty \frac{e^{-t} sin2t}{t} dt$, using Laplace transforms 6

C Using Convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ 8

Q5

A Find $L\{e^{-4t} sin3t\, cos2t\}$ 6

B Prove that the vector field $\bar{F}$ on R^3 defined by 6

$$\bar{F} = (y^2 cosx + z^3)\hat{\imath} + (2ysinx - 4)\hat{\jmath} + (3xz^2 + 2)\hat{k}$$

is conservative and find its scalar potential.

C Find the Fourier Series for $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $0 \le x \le 2\pi$ 8

Q6

A Obtain Fourier series expansion of $f(x) = 4 - x^2$ in (-2, 2) 6

B Prove that the matrix A is diagonalisable, find the transforming matrix and the diagonal matrix. 6

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

C

i) Find $L^{-1}\left\{\log\left(\frac{s^2+1}{s^2+2}\right)\right\}$ 4

ii) Find $L^{-1}\left\{\frac{1}{s^2+2s+10}\right\}$ 4